

## \* Digital control - Session

### Analysis

$$\text{clc eqn } |zI - A| = 0$$

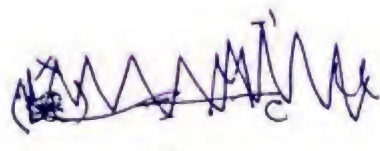
$$\text{T.F } C(zI - A)^{-1} B$$

### Controllability

→ اندر اسیج فی ار (Sys. states) ب (Control i/p)

$$M_c = [B \quad AB \quad A^2B \quad \dots \quad A^{n-1}B]$$

$$|M_c| \neq 0$$

→ control input 

$$x(n) = M_c^{-1} u$$

→ لازم یکنه قیة  $M_c$  موجوده عشانه وطلع قیة.  
فتکره  $M_c^{-1}$  موجوده.

## observability

→ أشود (sys. states) من خلال مخرجها (measuring)

$$M_o = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

$$M_o \neq 0$$

$$\underline{\text{Ex}} \quad x(k+1) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{pmatrix} x(k) + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u(k)$$

$$y(k) = \begin{pmatrix} 1 & 0 & 1 \end{pmatrix} x(k)$$

clc eqn

$$|zI - A| = \begin{vmatrix} z & 0 & 0 \\ 0 & z & 0 \\ 0 & 0 & z \end{vmatrix} - \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{vmatrix} = 0$$

$$= \begin{vmatrix} \oplus z & \ominus 0 & \oplus 0 \\ \ominus 0 & \oplus z & \ominus 1 \\ \oplus 1 & \ominus 2 & \oplus z+3 \end{vmatrix}$$

$$|ZI - A| = Z \begin{vmatrix} Z & -1 \\ 2 & Z+3 \end{vmatrix} = (-1) \begin{vmatrix} 0 & -1 \\ 1 & Z+3 \end{vmatrix} = 0$$

$$Z^3 + 3Z^2 + 2Z + 1 = 0$$

Controllability  $\mathcal{M}_c = (B \quad AB \quad A^2B)$

$$AB = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix}$$

$$A^2B = A \cdot AB = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 7 \end{bmatrix}$$

$$\mathcal{M}_c = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -3 \\ 1 & -3 & 7 \end{bmatrix}$$

$$|\mathcal{M}_c| = 1 \begin{vmatrix} 0 & 1 \\ 1 & -3 \end{vmatrix} = -1$$

observability  $M_o = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix}$

$$C = (1 \quad 0 \quad 1)$$

$$CA = (1 \quad 0 \quad 1) \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{pmatrix}$$

$$CA^2 = (1 \quad 0 \quad 1) \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{pmatrix} = (3 \quad 5 \quad 8)$$

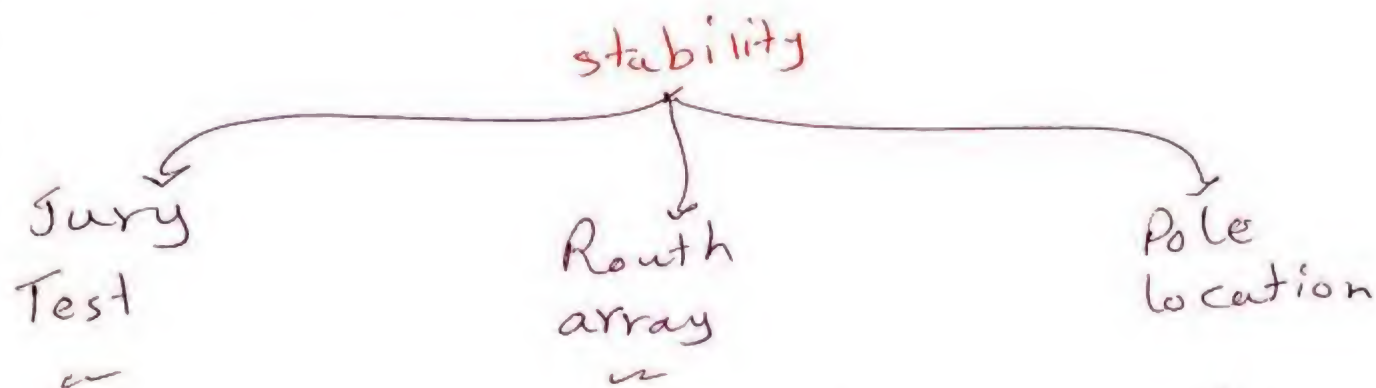
$$M_o = \begin{pmatrix} 1 & 0 & 1 \\ -1 & -1 & -3 \\ 3 & 5 & 8 \end{pmatrix}$$

$$|M_o| = \begin{vmatrix} 1 & 0 & 1 \\ -1 & -1 & -3 \\ 3 & 5 & 8 \end{vmatrix} = 1 \begin{vmatrix} -1 & -3 \\ 3 & 8 \end{vmatrix} - 1 \begin{vmatrix} -1 & -1 \\ 3 & 5 \end{vmatrix} = -8 - (-2) = -6$$



clc eqn

$$Z^3 + 3Z^2 + 2Z + 1 = 0$$



$$A: \text{system matrix} \rightarrow A_{\text{diagonal}} = T^{-1} A T$$

$$\begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

$\lambda \rightarrow$  eigen values

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$$\underline{T.F} \quad \frac{C \operatorname{adj} (Z I - A) B}{(Z I - A)}$$

$$|ZI - A| = \begin{vmatrix} Z & -1 & 0 \\ 0 & Z & -1 \\ 1 & 2 & Z+3 \end{vmatrix}$$

$$\text{adj}(ZI - A) = \begin{bmatrix} \oplus & \ominus & \oplus \\ \ominus & \oplus & \ominus \\ \oplus & \ominus & \oplus \end{bmatrix}^T$$

$$= \begin{bmatrix} Z^2 + 3Z + 2 & 1 & -Z \\ -Z - 3 & Z^3 + 3Z & 2Z + 1 \\ 1 & -Z & Z^2 \end{bmatrix}^T$$

كل حد يشكك فيه وعمود وأجيب المصداق الذي يطالع  
وبعد من الإشارة في خطوة ثانية.

$$= \begin{bmatrix} Z^2 + 3Z + 2 & -1 & -Z \\ Z + 3 & Z^3 + 3Z & -2Z - 1 \\ 1 & Z & Z^2 \end{bmatrix}^T$$

$$\text{adj}(zI - A) * B = \begin{bmatrix} z^2 + 3z + 2 & z + 3 & 1 \\ -1 & z^2 + 3z & z \\ -z & -2z - 1 & z^2 \end{bmatrix} * \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ z \\ z^2 \end{bmatrix}$$

$$C \text{adj}(zI - A) B = (1 \ 0 \ 1) \begin{bmatrix} 1 \\ z \\ z^2 \end{bmatrix} = 1 + z^2$$

$$T.F = \frac{z^2 + 1}{z^3 + 3z^2 + 2z + 1}$$

## Time response

$$\phi(z) = (zI - A)^{-1}$$

$$x(z) = \phi(z) * z * x(0) + \phi(z) B u(z)$$

$$x(k) = z^{-1} [x(z)]$$

$$y(k) = C x(k) + D u(k)$$

Ex

$$x(k+1) = \begin{pmatrix} 0.6 & 1 \\ 0 & 0.7 \end{pmatrix} x(k) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u(k)$$

$$y(k) = \begin{pmatrix} 1 & 1 \end{pmatrix} x(k) + 2 u(k)$$

$$x(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix} ; x(k) = 1$$

$$\phi(z) = (zI - A)^{-1} = \left[ \begin{pmatrix} z & 0 \\ 0 & z \end{pmatrix} - \begin{pmatrix} 0.6 & 1 \\ 0 & 0.7 \end{pmatrix} \right]^{-1}$$

$$= \begin{pmatrix} z + 0.7 & 1 \\ 0 & z - 0.6 \end{pmatrix} \frac{1}{(z - 0.6)(z - 0.7)}$$



$$X(z) = \frac{z}{(z-0.6)(z-0.7)} \begin{bmatrix} z-0.7 & 1 \\ 0 & z-0.6 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} +$$

$$\frac{z}{(z-1)(z-0.6)(z-0.7)} \begin{bmatrix} z-0.7 & 1 \\ 0 & z-0.6 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \frac{z}{(z-0.6)(z-0.7)} \begin{bmatrix} 1 \\ z-0.6 \end{bmatrix} + \frac{z}{(z-1)(z-0.6)(z-0.7)} \begin{bmatrix} 1 \\ z-0.6 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{z}{(z-0.6)(z-0.7)} \\ \frac{z}{z-0.7} \end{bmatrix} + \begin{bmatrix} \frac{z}{(z-1)(z-0.6)(z-0.7)} \\ \frac{z}{(z-1)(z-0.6)} \end{bmatrix}$$

كما نرى هنا تأخذ المصفوفة مصطلح مشترك و تجميع

الكسور ثم تدخل المصفوفة وتقل (Partial Fraction)

لكن (P.F) مش بيتعمل خارج المصفوفة.

مع اما تقل (P.F) لاير تخرج ال (z) برة ال P.F

$$\text{ex } z \left[ \frac{A_1}{z+a} + \frac{A_2}{z+b} \right]$$

$$X(z) \sim \left[ \frac{-10z}{z-0.6} + \frac{10z}{z-0.7} \right] + \left[ \frac{8.33z}{z-1} + \frac{25z}{z-0.6} - \frac{33.33z}{z-0.7} \right]$$

$$\left[ \frac{3.33z}{z-1} - \frac{3.33z}{z-0.7} \right]$$

$$X(k) \sim \begin{bmatrix} -10(0.6)^k + 10(0.7)^k \\ (0.7)^k \end{bmatrix} + \begin{bmatrix} 8.33 + 25(0.6)^k - 33.33(0.7)^k \\ 3.33 - 3.33(0.7)^k \end{bmatrix}$$

$$\sim \begin{pmatrix} 15(0.6)^k - 23.33(0.7)^k + 8.33 \\ 3.33 - 2.33(0.7)^k \end{pmatrix}$$

$$y(k) \sim (1) \quad 1) \quad X(k) + 2$$

$$\sim 15(0.6)^k + 23.33(0.7)^k + 8.33 + 3.33 -$$

$$2.33(0.7)^k + 2$$

$$y(k) \sim 13.66 + 15(0.6)^k - 25.66(0.7)^k$$